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# Predicting Conformal Aperture Directivity for arbitrary curved surfaces

T. Pelham, G. Hilton, E. Mellios, R. Lewis

Conformal Antenna Arrays offer many advantages for aerospace platforms, especially if selected early in the platform design cycle. To this end a Conformal antenna array directivity equation is presented, allowing the benefits of a conformal antenna array to be explored and design trade-offs considered at an early stage. The Conformal Directivity equation is an extension of Hannan's equation for the maximum achievable directivity of a planar aperture within an infinite antenna array, and when compared to a uniform illumination model for a planar aperture for aperture areas above  $0.5\lambda^2$  there is a maximum difference of 0.87dBi, and a mean difference of 0.49dBi.

**Introduction:** Conformal Antenna Arrays are commonly used in Defence and Aerospace arena, when a phased array antenna is required but there is no suitable surface for a planar antenna array. Conformal Antenna Arrays offer reduced aerodynamic drag compared to a planar array with a radome, in addition to a wider field of regard [1]. However these benefits come at the cost of increased design costs and technical risk factor. In an effort to reduce the cost of conformal antenna arrays, a practical way to assess the expected performance of an aperture is required, allowing a realistic assessment of the performance implications of a conformal array in comparison to a planar array with its inherent platform restrictions and penalties. In other sectors, such as medical imaging [2], conformal antenna arrays offer many advantages if the barrier to use in terms of design costs could be reduced.

The ability to predict the performance implications of the surface geometry without an in depth design study would reduce the design risk, and in the initial system concept design for any system that requires a sensor or communications aperture, would allow the trade-offs between aperture geometry and other system characteristics, such as aerodynamic profile to be assessed. The study has been focused on the more relevant geometries for aerospace systems, planar apertures, singly curved surfaces such as would be expected for the side of the fuselage of an aircraft, and paraboloid surfaces, such as are found on the leading edges of wings, and control surfaces, and in many nosecone profiles.

**Directivity of a Conformal Aperture:** Hannan [3] proposed a formula for the maximum achievable directivity of a planar antenna array, and its variation with observation angle, treating any planar array as a subset of an infinite planar array, and the broadside case ( $\theta = 0$ ) is easily recognisable.

$$D_{max}(\theta, \phi) = \frac{4\pi A}{\lambda^2} \cos\theta \quad (1)$$

$$D_{max}(\theta, \phi) = \frac{4\pi}{\lambda^2} \sum_{i=1}^N (A_i \cos(\theta + \alpha_i)) + \quad (2)$$

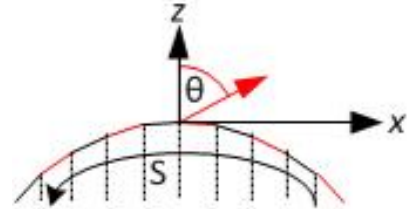
In Eq 2,  $A_i$  represents the area of the element or sub-array area, and  $\alpha_i$  is the angle between the element and the plane of the array. In this way Eq 2 gives the projection of the conformal array onto a plane defined in the same way as Eq 1. Indeed in this way Eq 2 can be considered as the general case, which collapses to Eq 1 for a planar antenna array.

This study proposes that the maximum obtainable directivity of any conformal antenna array on an arbitrary surface is directly related to the projection of its area at the observation angle of interest, extending Hannan's equation to a general surface. When a conformal surface is the only aperture available for a system antenna array, this function allows convenient estimation of the maximum achievable directivity, and the implications of small changes in the surface geometry.

**Uniform Illumination Function:** A method was sought to test the predictions of the Conformal Aperture Directivity Equation (Eq 2), and to this end a uniform illumination function has been proposed. In this model, a mesh of cartesian grid points are used to describe any surface of interest. These are uniformly spaced across the surface of the defined aperture, as shown in Figure 1 for a cylindrical aperture. The red and black segments represent the surface mesh. The arc length ( $s$ ) of the aperture is used to generate the grid points, separated by a constant fraction of the arc length. Not shown is the  $Y$  axis parameter for this aperture, which is height.

This mesh is then used to generate a series of Electric field vectors, representing a uniform field across the aperture. This arrangement of cartesian points allows decomposition of the electric field into a series of  $x$  and  $z$  components, which can then be summed and used to generate a far-field pattern using the Huygens transform, similar to some previous approaches [4].

In addition, a shadowing function is used based on the defined cartesian mesh, to calculate which points on the surface are visible for each  $(\theta, \phi)$  coordinate. This enhances the functionality of the model from a mesh of field vectors with no structure to obstruct them, to that of an approximation of an aperture integrated into a surface. In this way, the uniform illumination model allows a comparison with the Planar (Eq 1), and Conformal Aperture Directivity Equations (Eq 2) for a variety of generalised surfaces, which for this study will be specified in terms of wavelengths.

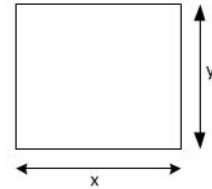


**Fig. 1** Conformal Mesh and X-Z Plane Parameters for a cylindrical conformal surface, arc length ( $s$ ), and radius ( $r$ )

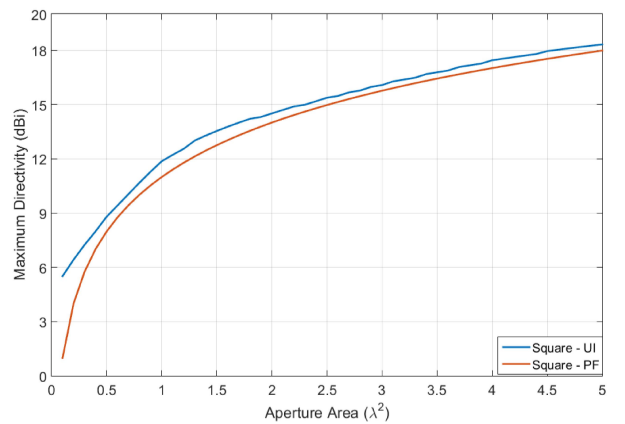
*Arbitrary Geometries:*

## Planar Apertures

Planar apertures reflect the simplest geometry of an aperture, and are uniquely defined by their dimensions in terms of width and height, in this study associated with the  $x$  and  $y$  dimensions respectively as shown in Figure 2. Planar apertures offer the opportunity to compare the model for Uniform Illumination of an aperture with the established analytical formula (Eq 1).



**Fig. 2** Parameters for a Planar aperture in the X-Y Plane, height ( $y$ ), and width ( $x$ )

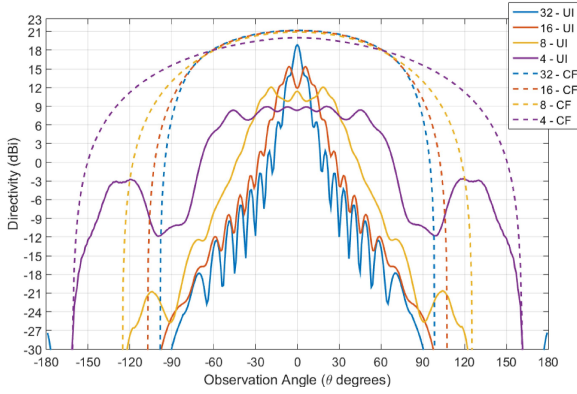


**Fig. 3** Comparison of Maximum Directivity for Square Apertures using Uniform Illumination (UI) and Planar Directivity Equation 1 (PF)

A comparison of the maximum directivity for squares and rectangles of the same area between the Uniform Illumination Model, and the planar directivity equation can be seen in Figure 3, the aperture's dimensions are  $\sqrt{\lambda^2}$ . There is good agreement between the two, with some divergence for very small apertures. The small disparity between the two methods is likely a reflection of the difference between the physical area of an aperture and the effective aperture. Balanis [5], defines the effective area or aperture of antenna or antenna array as "the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization matched to the antenna". In this situation, it is likely the absence of a fringing field term in the uniform illumination model which effectively enlarges the effective aperture beyond its defined limits.

### Cylindrical Apertures

Cylindrical Apertures are a very common structure, and a curved surface represents the simplest aperture structure that could be expected to be used as a conformal antenna array. A sub-cylindrical aperture is uniquely defined by the height, radius of curvature  $r$ , and by the arc length subtended by the aperture  $s$ . These are related by the angular range of the aperture in radians  $a$ , as  $s = ar$ , as shown in Figure 1. A comparison has been made between the predictions of the conformal directivity equation (Eq 2, CF), and the Uniform Illumination model (UI), in Figure 4. This figure shows the directivity predictions for a range of radii of curvature in wavelengths, for an aperture of arc length of  $10\lambda$ , and  $y$  dimension of  $1\lambda$ , for a consistent aperture area of  $10\lambda^2$ . It is interesting to note that at



**Fig. 4** Directivity with Observation Angle for different radii of curvature, and a consistent arc length of 10 wavelengths, comparing the conformal directivity equation (CF), and the uniform illumination function (UI).

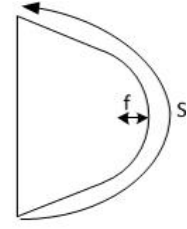
the Conformal Directivity Equation represents the directivity of an area, projected onto the observation plane, and can perhaps be thought of as the directivity envelope within which an array aperture can be expected to scan. Figure 4 clearly shows the expected progression between large radii of curvature to small. The directivity is reduced and the beamwidth increases, and this trend is clear from the Conformal Directivity equation and the Uniform Illumination model.

### Paraboloid Apertures

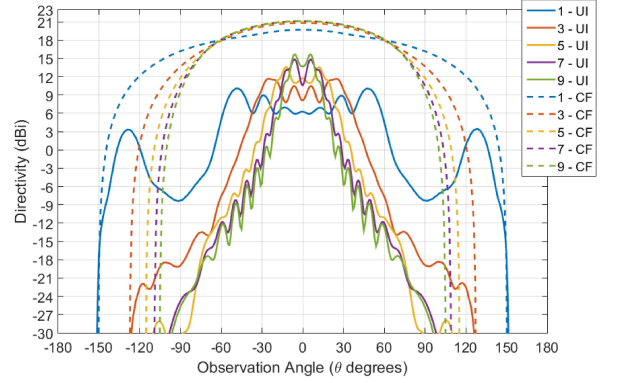
Parabolas, or similar structures such as ogives, are quite common, especially in the aerospace sector, as nosecones, or the leading edges of wings. Not as simple to define as circles, there is nonetheless a consistent formula to define the surface described, in terms of their focal length  $f$ , aperture height, and the arc length of the aperture  $s$ , as shown in Figure 5. As the arc length of a parabola between any two points on its surface may be defined in terms of their perpendicular distance from the axis of symmetry of the parabola ( $p_1, p_2$ ) and the focal length, with some additional conversion  $h_n = p_n/2$ ,  $q_n = \sqrt{f^2 + h_n^2}$ , and Eq 3.

$$s = \frac{h_1 q_1 - h_2 q_2}{f} + f \ln \left( \frac{h_1 + q_1}{h_2 + q_2} \right) \quad (3)$$

While the effects of reduction in radii for cylindrically curved aperture are interesting, following the proposal that a parabolic aperture represents the ideal geometry for consistent main beam directivity over a wide angular range, Figure 6 represents an investigation into this premise. Clearly, as the focal length is increased from  $1\lambda$ , representing a very



**Fig. 5** X-Z Plane parameters for a paraboloid surface, arc length ( $s$ ), and focal length ( $f$ )



**Fig. 6** Directivity with Observation Angle for Parabolas of different focal lengths, with a consistent arc length of 10 wavelengths, comparing the conformal directivity equation (CF), and the uniform illumination function (UI).

sharply curved aperture with a very linear profile in the region  $\pm 60^\circ$ . As the focal length increases to  $9\lambda$ , representing a much more gentle curve given the arc length of  $10\lambda$  the aperture directivity tends towards that of a planar plate of the same area as the focal length tends towards infinity, consistent with the predictions of the Conformal Aperture Directivity equation. This geometry is of particular interest in situations when the leading edge of a lifting surface is considered a desirable location for a sensor or communications aperture, and using this technique, the implications of the aerodynamic structure on the array performance can be easily examined. It is worth noting that this model does not consider the implications of phase offsets, and the zero phase weight 'broadside' pattern represents a non-optimal weighting for a  $0^\circ$  beam steering vector.

**Conclusion:** Over the range of aperture geometries considered here, the conformal aperture directivity equation demonstrates good agreement with the more complex uniform illumination model, for aperture areas greater than  $0.5\lambda^2$  the mean difference is 0.49dBi with a standard deviation of 0.18dBi for the maximum directivity predictions for the planar aperture. The maximum conformal directivity equation (Eq 2) should be considered like Hannan's planar maximum directivity equation as an envelope function, representing of a simple principle, if the projection of an apertures area onto the observation plane remains constant with changes in observation angle, then the maximum directivity remains unchanged.

While the design of an appropriate antenna element is critical to meet the antenna array performance goals, antenna array performance is strongly influenced by its geometry, and if the maximum scanning angle is a major design driver, then both the conformal aperture directivity equation and the uniform illumination model confirm expectations. If  $360^\circ$  of coverage is required, then a circular array is required, but if  $180^\circ$  is required, then a parabolic surface is ideal.

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